



## FUZZY GOAL PROGRAMMING APPROACH FOR SOIL ALLOCATION PROBLEM IN BRICK-FIELDS- A CASE STUDY

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**KEYWORDS:** Brick field, Brick production, Brick production planning, Fuzzy goal, Fuzzy goal programming.

### ABSTRACT

This paper presents fuzzy goal programming approach for optimal allocation of soil for Brick production planning for different types of Bricks in a Brick-field. Production of Bricks plays an important role in construction sector. Although construction sector is an unrecognized sector, it is steadily growing and contributing nation development. In order to survive and compete with other Brick fields, the decision-making unit of a Brick-field has to make decision that is competitive, practical, and challenging in order to meet the demand of the customers as well as markets. That is why it tries to maintain the quality of Bricks produced. In the model formulation, goals such as Brick production, net profit, water requirement, coal requirement, labor requirement, and machine utilization are considered as fuzzy as the decision makers prefer to describe it fuzzily to accommodate imprecise data. Then fuzzy goal programming approach is used to obtain the most satisfactory solution. To explore the application potential of the proposed model, the soil allocation planning problem of Kandakhola Brick-field at Santipur, Dist-Nadia and West Bengal, India is taken into consideration.

### INTRODUCTION

Brick Industry appears to be ancillary to the construction industry. The existence and future of brick industry is directly related to the development of the construction industry. The present production of burnt clay Bricks in India is estimated at about 120 billion Bricks per year producing from at least 1,00,000 Brickfields ( of which at least 40,000 are moving / fixed chimney / Hoffmann / High-draught kilns ) situated all over India. Indian present consumption factor is around 100.

Brick-field planning problems are important from both constructional and economic point of view. It involves a complex interaction between nature and economics. Since population increases steadily in third world countries like India, there exists a need of more production of Bricks for constructional field. One way of achieving high productivity is to increase soil allocation for various types of Bricks. Third world countries like India and other countries are losing land because of high population growth and industrialization. As a result production of Bricks must be increased by proper utilization of resources. Brick production planning depends on several resources like the availability of soil, water, labor, machine, capital, coal etc. It also requires soil characteristics, Brick quality, socio-economic conditions, climate and many other factors. Decision-makers of Brickfields usually use a wide range of production system, which results in large variations in Brick production. Brick-field planning problem, generally, involves multiple goals such as maximizing of Brick production, maximizing overall profit, minimizing labor expenditures, water requirement, coal requirement, machine utilization and others. These goals are conflicting in nature. It is not possible to obtain all the targets goals at the optimum level simultaneously. Certain goals may be achieved with the expense of others. A compromise among the conflicting goals is required to obtain a satisfactory solution in the decision making process. Goal programming (GP) is a useful tool for dealing with problems having multiple and conflicting goals subject to given system constraints of the problem. Several researchers like Ijiri [1], Lee [2], Goodman [3], Ignizio [4], and Romero [5] implemented goal programming approach in decision making problems. In conventional goal programming, parameters of the problem are precisely defined. For a Brickfield problem, values of some parameters may not be known precisely. They are rather defined in a fuzzy-sense. For successfully handling such problems, the fuzzy goal programming (FGP) technique may be used. FGP approach to soil allocation planning problem is yet to appear in the literature

The objectives of the paper are:

- i) to present FGP model for optimal allocation of soil for Brick production
- .ii) to propose an annual production plan for decision makers in Brick-field.



The soil allocation planning problem of Kandokhola Brick-field, Santipur, Dist-Nadia, West Bengal, India is considered here to demonstrate the potential use of the approach.

## METHODOLOGY

In 1980, Narasimhan [6] used the concept of fuzzy set theory in GP by incorporating fuzzy goals and constraints within the traditional GP model. Hannan [7] developed an alternative FGP model to the Narasimhan's model [6]. Hannan [7] demonstrated how the FGP problem consisting of  $2^K$  linear programming (LP) problems, each containing  $3K$  constraints, may be reformulated as a single LP problems with only  $2K$  constraints, where  $K$  is the number of fuzzy goals in the original problem. Hannan [8] mentioned inconsistencies in Narashimhan's fuzzy priorities and proposed that either Saaty's approach [9] or that of Zeleny [10] could be used. Narasimhan [11] reexamined the general problem of GP with fuzzy priorities and proved the validity of the previously suggested approach and discussed the general problem of fuzzy priorities. Hannan [8] followed up the development and commented that "Narasimhan's method [11] requires that the decision maker compresses the acceptable goal interval in accordance with the (fuzzy) priorities of the goal rather than state (fuzzy) relative value of attaining the goal which does not constitute a direct way of dealing with fuzzy weights." Following the publication of Narasimhan's paper [11] and subsequent commentaries in the journal 'Decision Sciences', James P. Ignizio [12], an important contributor of GP criticized heavily two alternative methods for GP problem with fuzzy goals proposed by Narasimhan [6] and Hannan [7]. He was very much of the opinion that Zimmermann's method [13] published in 1978 offers an even more efficient approach than Hannan's approach [7] or particularly Narasimhan's approach [6]. He mentioned that Zimmermann had already developed the basic multi-criteria programming model without using deviational variables, which are according to him unnecessary in the basic minimax version of Zimmermann [13]. According to his opinion, triangular membership functions described by both Hannan and Narasimhan are highly questionable. He favored "ramp" membership function described by Zimmermann as Zimmermann's model yields results identical to Hannan's model [7]. He praised Zimmermann [13] for discussing the use of nonlinear membership functions that lead to fuzzy nonlinear multicriteria programming. He criticized both Hannan [7] and Narasimhan [6] for ignoring nonlinear model as the linear membership functions might be inadequate to deal with several problems. However, Hannan [14] believed that Narasimhan's paper [6] offered a unique contribution and is not merely "reinventing the wheel" as described by Ignizio [12]. Hannan [14] pointed out that the difference between Narasimhan's fuzzy goal programming formulation [6] and Zimmermann's fuzzy multi-criteria formulation [13] is significantly the underlying philosophy of decision maker's input. Despite Ignizio's criticism [12], researchers show interests in Hannan's model [14]. Rubin and Narasimhan [15] established a methodology based on the use of a nested hierarchy of priorities for each goal. Tiwari et al. [16] studied how the preemptive structure can be used in FGP problems. Their procedure is consistent with the preemptive priority structure of the decision maker and it reduces the number of sub problems with respect to the approach by Narasimhan [6]. Tiwari et al. [17] presented an alternative additive model for maximizing the membership function directly for FGP in 1987. Rao et al. [18] introduced the concept of relative flexibility in FGP. They used the concept of pair-wise comparison method in the sense of Shimura [19] to determine the relative flexibility of the goals realized under a fixed set of aspiration levels in fuzzy environment. They computed the aggregated relative flexibility for all goals by using Sherali's algorithm [20]. Chen [21] proposed an algorithm for solving a FGP problem with symmetrically triangular membership functions for fuzzy goals and priority structure and showed the efficiency of the algorithm by justifying computational superiority over the procedure proposed by Tiwari et al. [17] in 1994. For modeling imprecise goals, Martel and Aouni [22, 23, 24] and Aouni et al. [25] reformulated the standard GP model of Charness and Cooper [26] by introducing the satisfaction degrees as a function of the goal deviations in the objective function of the model. In 1992, Mohamed [27] presented a chance constrained FGP by using the concept of the conventional GP in which he considered the problem for achievement of each of the membership functions to its highest value (unity) by minimizing the deviational variables of the corresponding membership goals. In 1997, Mohamed [28] further studied the relationship between GP and fuzzy programming. Kim and Whang [29] investigated the application of tolerance concepts to GP in fuzzy environment. They extended their methodology to accommodate FGP model of Hannan [7] with the unbalanced linear membership functions and preemptive priorities. In 2001, Chen and Tsai [30] further discussed FGP with different importance and priorities in which they formulated FGP model based on piecewise linear approximation suggested by Yang et al. [31]. Abd El-Wahed and Abo-Sinna [32] proposed a hybrid FGP solution method to determine weights and weights to the objectives (sub-objectives) under different (same) priorities using the concepts of fuzzy membership functions along with the notion of degree of conflict among objectives. The authors claim that both linear and nonlinear problems can be solved by hybrid FGP method. The modeling aspects of FGP within the framework of conventional GP have been further studied by Hannan [14], Kuwano [33], Mohanty and



Vijayadraghavan [34], Ramik [35], Rao et al. [18, 36], Lin [37] and others. Pal and Moitra [38] used the Mohamed’s FGP model [28] for solving problems with multiple fuzzy goals using dynamic programming. In the solution process, Mohamed used  $\mu - d^- + d^+ = 1$ , where  $\mu$  is the membership function,  $d^-$  and  $d^+$  are negative and positive deviational variables. Since maximum value of membership function is 1, positive deviation is not possible at all. Since  $d^+ \geq 0$ , Pramanik and Roy [39, 40, 41, 42, 43] use  $\mu - d^+ \geq 1$ . However,  $d^+ = 0$ , so Pramanik and Dey [44] used  $\mu - d^- = 1$  for all cases. So, positive deviation is not required in the model formulation. The model formulation becomes simple and easy to solve. Since positive deviation is absent in the model formulation, the computational load is less than Mohamed’s model [28]. Gupta and Bhattacharjee [45] studied Hannan’s FGP model [7] by omitting positive deviational variables. Implementation of FGP approach is found in the research work of Pramanik [46], Banerjee and Pramanik [47], Dey et al. [48, 49] and other researchers.

In this study, we use FGP approach proposed by Pramanik and Roy [42] and Pramanik and Dey [44] to modeling the problem. In order to formulate soil allocation planning for a year, total time is divided into two seasons according to climate conditions. Notations used to formulate the FGP model of the problem are defined in Table 1.

Table 1 Description of notations

Notations	Explanations
b	Index for the Brick $b \in \{1,2,3,\dots,B\}$
s	Index for the season $s \in \{1,2\}$
V	1000 Cube feet
$X_{bs}$	Volume of soil used for Brick b in season s
$L_s$	Total soil used for all Bricks in season s
$P_{bs}$	Average production of Brick b per unit volume of soil in season s
$T_{bs}$	Total production target of Brick b (numbers) in season s
$L_{bs}$	Labor requirement per unit volume of soil for Brick b in season s
$TL_s$	Expected labor availability in the season s (man-days)
$I_{bs}$	Average investment per unit volume of soil of Brick b in season s
$TI_s$	Total investment available in season s
$M_{bs}$	machine hours per unit volume of soil for Brick b in season s
$TM_s$	Expected total machine hours available in season s
$N_{bs}$	Average profit for per unit volume of soil for Brick b in season s
$N_s$	Expected net profit for all Bricks in season s
$W_{bs}$	Average amount of water requirements for per unit volume of soil of Brick b in season s
$W_s$	Expected total ground water available in season s
$CO_{bs}$	Average amount of coal requirement for per unit volume of soil of Brick b in season s
$CO_s$	Expected coal availability in season s
$E_{bs}$	Average amount of miscellaneous expenditure for per unit volume of soil of Brick b in season s



$E_s$	Expected expenditure in season $s$
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## THE DESCRIPTION OF THE GOALS

### The description of the goals

The goals for the FGP problem may be defined as follows:

#### 3.1 Brick production goal

The decision maker always tries to maximize expected Brick production. This is obtained by multiplication of soil used for Brick  $b$  in season  $s$  with average production per unit volume (soil) of Brick. The sum of the productions for all Bricks should be greater or equal to the expected production target during the year. The fuzzy goal constraints for Brick production can be expressed as follows:

$$\sum_{b=1}^B P_{bs} X_{bs} \geq \sum_{b=1}^B T_{bs}, \text{ where } s = 1, 2$$

$$\Rightarrow Z_{1s} \geq b_{1s}, \text{ where } Z_{1s} = \sum_{b=1}^B P_{bs} X_{bs} \text{ and } b_{1s} = \sum_{b=1}^B T_{bs}, s = 1, 2$$

#### 3.2 Net profit goal

The decision maker always tries to maximize profit from the Brick production. The fuzzy goal constraint for net profit can be expressed as follows:

$$\sum_{b=1}^B N_{bs} X_{bs} \geq N_s, \text{ where } s = 1, 2$$

$$\Rightarrow Z_{2s} \geq b_{2s}, \text{ where } Z_{2s} = \sum_{b=1}^B N_{bs} X_{bs} \text{ and } b_{2s} = N_s, s = 1, 2$$

#### 3.3 Water requirement goal

To meet the production target of Bricks in a year, water supply must be ensured. The fuzzy goal constraint for water supply can be written as follows:

$$\sum_{b=1}^B W_{bs} X_{bs} \leq W_s, \text{ where } s = 1$$

$$\Rightarrow Z_{3s} \leq b_{3s} \text{ where } Z_{3s} = \sum_{b=1}^B W_{bs} X_{bs} \text{ and } b_{3s} = W_s$$

#### 3.4 Labor requirement goal

In general, the administration of the Brick field hires estimated number of labors throughout the year in order to smooth functioning of the production. The fuzzy goal constraint for labors may be written as follows:

$$\sum_{b=1}^B L_{bs} X_{bs} \leq TL_s, \text{ where } s = 1, 2$$

$$\Rightarrow Z_{4s} \leq b_{4s} \text{ where } Z_{4s} = \sum_{b=1}^B L_{bs} X_{bs} \text{ and } b_{4s} = TL_s, s = 1, 2$$

#### 3.5 Machine utilization goal

For preparing proper mixture of soil and water throughout the year, there is an annual machine hour estimate. The machine hour allocated to each season should not exceed the machine hours available in each season. The fuzzy goal constraints for annual machine hours can be expressed as:

$$\sum_{b=1}^B M_{bs} X_{bs} \leq TM_s, \text{ where } s = 1$$

$$\Rightarrow Z_{5s} \leq b_{5s}, \text{ where } Z_{5s} = \sum_{b=1}^B M_{bs} X_{bs} \text{ and } b_{5s} = TM_s$$

#### 3.6 Coal requirement goal

For Brick production from raw Bricks, it is necessary to burn the raw Bricks properly. So coal is necessary throughout the year. The fuzzy goal constraints for annual coal requirement can be presented as follows:

$$\sum_{b=1}^B CO_{bs} X_{bs} \leq CO_s, \text{ where } s = 1, 2$$



$$\Rightarrow Z_{6s} \leq b_{6s}, \text{ where } Z_{6s} = \sum_{b=1}^B CO_{bs} X_{bs} \text{ and } b_{6s} = CO_s$$

**SOIL AVAILABILITY AND WORKING CAPITAL REQUIREMENTS**

**Soil availability and working capital requirements**

Decision making unit of the Brick-field fixes resources like available soil and available budget in order to meet different essential requirements. It tries to ensure maximum number of quality Bricks.

To satisfy essential requirements, the following constraints must be satisfied.

**4.1 Soil availability**

For production of each red Brick, soil is necessary for each season. . The total used soil for production of all Bricks for season s must not exceed the sum of available soil in the season s. The goal constraints for used soil can be expressed as follows:

$$\sum_{b=1}^B X_{bs}, \text{ where } s = 1, 2.$$

**4.2 Working capital requirement**

The goal constraints for the working capital throughout the year can be written as follows:

$$\sum_{b=1}^B I_{bs} X_{bs} \leq \sum_{s=1}^2 TI_s, \text{ where } s = 1,2$$

**4.3 Miscellaneous expenditure**

The Brick-field expends a certain amount of money for transportation, purchasing of parts of car, medicine for labors, electricity, diesel, kerosene, donation for festivals, etc. In the present study, these expenditures are considered as miscellaneous expenditure. The goal constraint for the miscellaneous expenditure throughout the year can be written as follows:

$$\sum_{b=1}^B E_{bs} X_{bs} \leq E_s, \text{ where } s = 1,2$$

**FORMULATION OF FUZZY GOALS**

In the proposed FGP model of soil allocation problem for Brick-field, the Brick production goal and net profit goal are of the types  $z_{ks}(\bar{x}) \geq b_{ks}$ . On the other hand, labor requirement goal, water requirement goal, coal requirement goal, machine utilization goals are of the types  $z_{ks}(\bar{x}) \leq b_{ks}$ .

In fuzzy goal programming, the membership function corresponding to the k-th fuzzy goal of the type  $z_{ks}(\bar{x}) \geq b_{ks}$  (see the Fig. 1) is defined as follows:

$$\mu_{z_{ks}}(\bar{x}) = \left\langle \begin{array}{l} = 1, \text{ if } z_{ks}(\bar{x}) \geq b_{ks} \\ = \frac{(z_{ks}(\bar{x}) - (b_{ks} - t'_{ks}))}{t'_{ks}}, \text{ if } b_{ks} - t'_{ks} \leq z_{ks}(\bar{x}) \leq b_{ks} \\ = 0, \text{ if } z_{ks}(\bar{x}) < b_{ks} - t'_{ks} \end{array} \right\rangle, \text{ where } b_{ks} - t'_{ks} \text{ and } b_{ks} \text{ are the lower-tolerance}$$

limit and the upper-tolerance limit for the k-th fuzzy goal respectively.

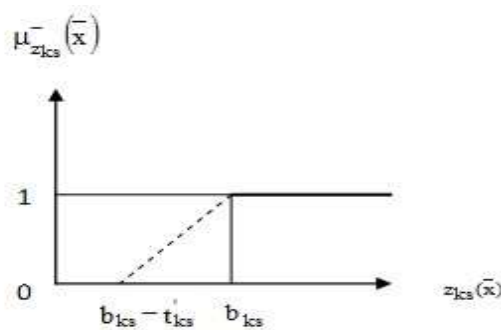


Fig.1 Membership function of the type  $z_{ks}(\bar{x}) \geq b_{ks}$



Similarly, membership function corresponding to the k-th fuzzy goal of the type  $z_{ks}(\bar{x}) \leq b_{ks}$  (see the Fig. 2) can be defined as:

$$\mu_{z_{ks}}(\bar{x}) = \begin{cases} = 1, & \text{if } z_{ks}(\bar{x}) \leq b_{ks} \\ = \frac{(b_{ks} + t_{ks}'' - z_{ks}(\bar{x}))}{t_{ks}'}, & \text{if } b_{ks} \leq z_{ks}(\bar{x}) \leq b_{ks} + t_{ks}'' \\ = 0, & \text{if } z_{ks}(\bar{x}) \geq b_{ks} + t_{ks}'' \end{cases}$$

Here  $b_{ks}$  and  $b_{ks} + t_{ks}''$  are the lower-tolerance and the upper-tolerance limit for the k-th fuzzy goal. Here,  $\mu_{z_{ks}}(\bar{x}) \in [0, 1]$ .

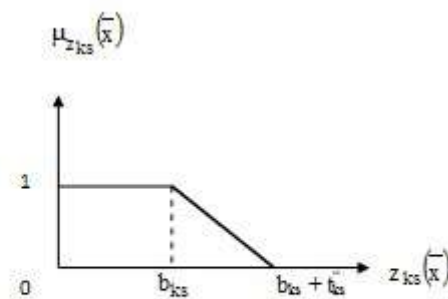


Fig.2 Membership function of the type  $z_{ks}(\bar{x}) \leq b_{ks}$

### FGP FORMULATION

The soil allocation planning problem can be presented in two seasons namely season1 and season2 for Brick-field. Since all three types Bricks are produced, restriction is imposed for each type.

#### 6.1 SEASON1:

Now following the proposed procedure the resulting executable model for season1 can be presented as follows:

$$\text{Min} (w_{11}d_{z_{11}}^- + w_{21}d_{z_{21}}^- + w_{31}d_{z_{31}}^- + w_{41}d_{z_{41}}^- + w_{51}d_{z_{51}}^- + w_{61}d_{z_{61}}^-) \quad (1)$$

subject to

$$\mu_{z_{11}} + d_{z_{11}}^- = 1$$

$$\mu_{z_{21}} + d_{z_{21}}^- = 1$$

$$\mu_{z_{31}} + d_{z_{31}}^- = 1$$

$$\mu_{z_{41}} + d_{z_{41}}^- = 1$$

$$\mu_{z_{51}} + d_{z_{51}}^- = 1$$

$$\mu_{z_{61}} + d_{z_{61}}^- = 1$$

$$\sum_{b=1}^3 X_{b1} \geq L_1$$

$$\sum_{b=1}^3 I_{b1} X_{b1} \leq TI$$

$$\sum_{b=1}^3 E_{b1} X_{b1} \leq E_1$$

$$X_{bs} \geq \alpha_{bs}, s=1, b=1, 2, 3$$

$$w_{11} + w_{21} + w_{31} + w_{41} + w_{51} + w_{61} = 1$$

$$0 \leq d_{z_{ks}}^- \leq 1, s=1, k=1, 2, \dots, 6$$



$$X_{bs} \geq 0, s=1, b=1, 2, 3$$

Here,  $w_{ks}$  are the weights associated with deviational variable  $d_{z_{ks}}^-$ .

By simplifying we can present the problem (1) as follows:

$$\text{Min} (w_{11}d_{z_{11}}^- + w_{21}d_{z_{21}}^- + w_{31}d_{z_{31}}^- + w_{41}d_{z_{41}}^- + w_{51}d_{z_{51}}^- + w_{61}d_{z_{61}}^-) \quad (2)$$

subject to

$$\frac{\sum_{b=1}^3 P_{bl} X_{bl} - \sum_{b=1}^3 T_{bl}}{t_{11}} + d_{z_{11}}^- = 1$$

$$\frac{\sum_{b=1}^3 N_{bl} X_{bl} - N}{t_{21}} + d_{z_{21}}^- = 1$$

$$\frac{w_1 - \sum_{b=1}^3 W_{bl} X_{bl}}{t_{31}} + d_{z_{31}}^- = 1$$

$$\frac{TI - \sum_{b=1}^3 L_{bl} X_{bl}}{t_{41}} + d_{z_{41}}^- = 1$$

$$\frac{TM - \sum_{b=1}^3 M_{bl} X_{bl}}{t_{51}} + d_{z_{51}}^- = 1$$

$$\frac{CO_1 - \sum_{b=1}^3 CO_{bl} X_{bl}}{t_{61}} + d_{z_{61}}^- = 1$$

$$\sum_{b=1}^3 X_{bl} \geq L_1$$

$$\sum_{b=1}^3 I_{bl} X_{bl} \leq TI$$

$$\sum_{b=1}^3 E_{bl} X_{bl} \leq E_1$$

$$X_{bs} \geq \alpha_{bs}, s=1, b=1, 2, 3$$

$$w_{11} + w_{21} + w_{31} + w_{41} + w_{51} + w_{61} = 1$$

$$0 \leq d_{z_{ks}}^- \leq 1, s=1, k=1, 2, \dots, 6$$

$$X_{bs} \geq 0, s=1, b=1, 2, 3$$

### 6.2 SEASON2:

In season 2, water requirement and machine hour are not included because rest of the produced raw bricks in season 1 are burnt in the second season. For the season 2 the problem can be formulated as follows:

$$\text{Min} (w_{12}d_{z_{12}}^- + w_{22}d_{z_{22}}^- + w_{42}d_{z_{42}}^- + w_{62}d_{z_{62}}^-) \quad (3)$$

subject to

$$\mu_{z_{12}} + d_{z_{12}}^- = 1$$

$$\mu_{z_{22}} + d_{z_{22}}^- = 1$$

$$\mu_{z_{42}} + d_{z_{42}}^- = 1$$

$$\mu_{z_{62}} + d_{z_{62}}^- = 1$$

$$\sum_{b=1}^3 X_{b2} \geq L_2$$

$$\sum_{b=1}^3 I_{b2} X_{b2} \leq TI$$

$$\sum_{b=1}^3 E_{b2} X_{b2} \leq E_2$$

$$X_{bs} \geq \alpha_{bs}, s=2, b=1, 2, 3$$

$$w_{12} + w_{22} + w_{42} + w_{62} = 1$$



$$0 \leq d_{zks}^- \leq 1, \quad s = 2; k = 1, 2, 4, 6,$$

$$X_{bs} \geq 0, \quad s = 2; b = 1, 2, 3$$

Here,  $w_{ks}$  are the weights associated with deviational variable  $d_{zks}^-$ .

By simplifying we can present the problem (1) as follows:

$$\text{Min} (w_{12}d_{z12}^- + w_{22}d_{z22}^- + w_{42}d_{z42}^- + w_{62}d_{z62}^-) \quad (4)$$

subject to

$$\frac{\sum_{b=1}^3 P_{b2} X_{b2} - \sum_{b=1}^3 T_{b2}}{t_{12}} + d_{z12}^- = 1$$

$$\frac{\sum_{b=1}^3 N_{b2} X_{b2} - N}{t_{22}} - d_{z22}^- = 1$$

$$\text{TI} - \frac{\sum_{b=1}^3 L_{b2} X_{b2}}{t_{42}} + d_{z42}^- = 1$$

$$\frac{\text{CO}_2 - \sum_{b=1}^3 \text{CO}_{b2} X_{b2}}{t_{62}} + d_{z62}^- = 1$$

$$\sum_{b=1}^3 X_{b2} \geq L_2$$

$$\sum_{b=1}^3 I_{b2} X_{b2} \leq \text{TI}$$

$$\sum_{b=1}^3 E_{b2} X_{b2} \leq E_2$$

$$X_{bs} \geq \alpha_{bs}, \quad s = 2, b = 1, 2, 3$$

$$w_{12} + w_{22} + w_{42} + w_{62} = 1$$

$$0 \leq d_{zks}^- \leq 1, \quad s = 2; k = 1, 2, 4, 6,$$

$$X_{bs} \geq 0, \quad s = 2; b = 1, 2, 3$$

## EUCLIDEAN DISTANCE

The real challenge is how to compare the solutions obtained from different methods or approaches. One simple way for comparison is to use Euclidean distance function. Yu [50] studied the concept of utopia point (ideal point) and used the distance function for group decision analysis. Since the aspired level of each membership goal is unity, the point consisting of the highest membership value of each goal would represent the ideal point. The Euclidean distance function Zeleny [51] can be defined as follows:

$$D = \left[ \sum_{i=1}^n (1 - \mu_n(\bar{x}))^2 \right]^{\frac{1}{2}}$$

Here,  $\mu_n(\bar{x})$  is the membership value for the solution  $\bar{x}$ . Now the solution for which D is minimal would be the most satisfactory solution.

## A CASE STUDY

The study was conducted at the Brick-field namely, Kandokhola Brick-field, Santipur, Nadia District, West Bengal, India for financial year 2012-2013. The village Kandokhola is situated at 5 km far from Santipur station. The village lies in the Nadia district. The approximate area of the Brick field is about 360000 square feet. Maximum Bricks are supplied from the field among the neighborhood areas. There are three types of quality Bricks like Brick-1, Brick-2, and Picket. For maximum production of Brick-1, it is very essential to allocate maximum amount of loam Soil. Similarly for Brick-2 and picket it is essential to allocate sandy soil and clay soil respectively.

The data regarding the production of Bricks (number/V), soil used (V), water consumption (liter), coal requirement (quintal), labor requirement (man-days/V), machinery hours (hrs/V) requirement for all types of Bricks throughout the year have been collected from various sources (Brick-field calculation book, field manager,





labor’s opinion etc.). The types of Bricks are denoted by  $b = 1$  for Brick-1,  $b = 2$  for Brick-2 and  $b = 3$  for Pickets. The first season ( $s = 1$ ) is defined by the period from September to April and the second season ( $s = 2$ ) is defined by the period from May to August. Total soil used for Bricks are more than 3500V (3000V for season1 and 500V for season2). Average purchasing price of soil is Rs.800/1000V. Total investment throughout the year is Rs 82000000.00 (Rs 80000000.00 for season1 and Rs200000.00 for season2). Total miscellaneous expenditure throughout the year is not more than Rs 1500000.00 (Rs 1000000.00 for season1 and Rs 500000.00 for season2). For both seasons, the required data are summarized in the following table 2, table 3, table 4 and table5.

**Table 2 The data description of fuzzy goals and their tolerances**

Goal	Aspiration level	Tolerance
Production in season 1(number)	3000000	2000000
Production in season 2(number)	500000	300000
Net Profit in season 1(Rs)	7500000	6500000
Net Profit in season 2(Rs)	145000	45000
Labor Requirement in season 1(Man/Day)	8000	6500
Labor Requirement in season 2(Man/Day)	300	100
Machine utilization(Hour)	2300	1900
Water Requirement(Liter)	57700	45700
Coal Requirement in season1 (Quintal)	7000	2000
Coal Requirement in season2 (Quintal)	240	60

**Table 3 Data description of per unit V for Season 1, V = 1000 cube feet**

Bricks	$M_{bs}$ (hr/V)	$L_{bs}$ ( man- days/V)	$W_{bs}$ (liter/V)	$P_{bs}$ (number/V)	$I_{bs}$ (Rs/V)	$N_{bs}$ (Rs/V)	$CO_{bs}$ (quintal/V)	$E_{bs}$ (Rs/V)
Brick-1	0.6	2.03	14.5	10100	25120	2000	2.1	157
Brick-2	0.55	2.03	14.3	10000	24925	2002	2.1	148
Picket	0.55	1.93	14.0	9900	24430	2005	2.2	144

**Table 4 Data description of per unit V for Season-2, V= 1000 cube feet**

Bricks	$M_{bs}$ (hr/V)	$L_{bs}$ ( man- days/V)	$W_{bs}$ (liter/V)	$P_{bs}$ (number/V)	$I_{bs}$ (Rs/V)	$N_{bs}$ (Rs/V)	$CO_{bs}$ (quintal/V)	$E_{bs}$ (Rs/V)
Brick-1	0.00	0.57	0.00	10000	3580	285	0.45	60
Brick-2	0.00	0.57	0.00	9900	3575	280	0.45	58
Picket	0.00	0.57	0.00	9800	3570	275	0.46	56

**Table 5 Description of the variables and membership functions of the model**

$X_{11}$	Soil used for Brick-1 in season 1	$\mu_{z_{21}}^-$	Membership grade for profit goal in season 1
$X_{12}$	Soil used for Brick-1 in season 2	$\mu_{z_{22}}^-$	Membership grade for profit goal in season 2
$X_{21}$	Soil used for Brick-2 in season 1	$\mu_{z_{31}}^-$	Membership grade for water requirement goal in season 1
$X_{22}$	Soil used for Brick-2 in season 2	$\mu_{z_{41}}^-$	Membership grade for labor requirement goal in season 1
$X_{31}$	Soil used for Picket in season 1	$\mu_{z_{42}}^-$	Membership grade for labor requirement goal in season 2
$X_{32}$	Soil used for Picket in season 2	$\mu_{z_{51}}^-$	Membership grade for machine utilization goal in season 1
$\mu_{z_{11}}^-$	Membership grade for production goal in season 1	$\mu_{z_{61}}^-$	Membership grade for coal requirements goal in season 1



$\mu_{z12}^-$	Membership grade for production goal in season 2	$\mu_{z62}^-$	Membership grade for coal requirements goal in season 2
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## RESULTS

The FGP model is formulated using the collected data (see the table 2). Here, all the goals lie in the same priority level. For both seasons, the tolerances values are provided in the table 2.

### 9.1 FGP formulation for season-1

$$\text{Min } (d_{z11}^- + d_{z21}^- + d_{z31}^- + d_{z41}^- + d_{z51}^- + d_{z61}^-) / 6$$

Subject to

$$(1010 x_{11} + 1000 x_{21} + 990 x_{31} - 1000000) / (3000000 - 1000000) + d_{z11}^- = 1$$

$$(2000 x_{11} + 2002 x_{21} + 2005 x_{31} - 1000000) / (7500000 - 1000000) + d_{z21}^- = 1;$$

$$(8000 - (2.03 x_{11} + 2.03 x_{21} + 1.93 x_{31})) / (8000 - 1500) + d_{z31}^- = 1;$$

$$(57700 - (14.5 x_{11} + 14.3 x_{21} + 14 x_{31})) / (57700 - 12000) + d_{z41}^- = 1;$$

$$(2300 - (0.6 x_{11} + 0.55 x_{21} + 0.55 x_{31})) / (2300 - 400) + d_{z51}^- = 1;$$

$$(7000 - (2.1 x_{11} + 2.1 x_{21} + 2.2 x_{31})) / (7000 - 5000) + d_{z61}^- = 1;$$

$$x_{11} + x_{21} + x_{31} \geq 3000;$$

$$25120 x_{11} + 24925 x_{21} + 24430 x_{31} \leq 80000000;$$

$$157 x_{11} + 148 x_{21} + 144 x_{31} \leq 10000000;$$

$$x_{11} \geq 1020;$$

$$x_{21} \geq 900;$$

$$x_{31} \geq 1080;$$

$$0 \leq d_{zi1}^- \leq 1, \quad i = 1, 2, \dots, 6$$

$$x_{i1} \geq 0, \quad i = 1, 2, 3$$

### 9.2 FGP formulation for season-2

$$\text{Min } (d_{z12}^- + d_{z22}^- + d_{z42}^- + d_{z62}^-) / 4$$

subject to

$$1000 x_{12} + 990 x_{22} + 980 x_{32} - 2000000) / (5000000 - 2000000) + d_{z12}^- = 1;$$

$$(285 x_{12} + 280 x_{22} + 275 x_{32} - 1000000) / (145000 - 100000) + d_{z22}^- = 1;$$

$$(300 - (0.57 x_{12} + 0.57 x_{22} + 0.57 x_{32})) / (300 - 200) + d_{z42}^- = 1;$$

$$(240 - (0.45 x_{12} + 0.45 x_{22} + 0.46 x_{32})) / (240 - 180) + d_{z62}^- = 1;$$

$$x_{12} + x_{22} + x_{32} \geq 500;$$

$$3580 x_{12} + 3575 x_{22} + 3570 x_{32} \leq 20000000;$$

$$60 x_{12} + 58 x_{22} + 56 x_{32} \leq 5000000;$$

$$x_{12} \geq 250;$$

$$x_{22} \geq 150;$$

$$x_{32} \geq 80;$$

$$0 \leq d_{zi2}^- \leq 1, \quad i = 1, 2, 4, 6;$$

$$x_{i2} \geq 0, \quad i = 1, 2, 3$$

$$x_{12} \geq 250;$$

Note 1: Soil allocation and goal achievement values corresponding to the normalized weighting structure are provided in the table 6.

**Table 6: Soil allocation and obtained membership values under proposed FGP approach**

Season1		
Min $\lambda$	( $x_{11}, x_{21}, x_{31}$ )	( $\mu_{z11}^-, \mu_{z21}^-, \mu_{z31}^-, \mu_{z41}^-, \mu_{z51}^-, \mu_{z61}^-$ )
2.98176	1020, 900, 1080	0.9997, 0.7703385, 0.3104615, 0.326477, 0.3152632, 0.296
Season2		
Min $\lambda$	( $x_{11}, x_{21}, x_{31}$ )	( $\mu_{z11}^-, \mu_{z21}^-, \mu_{z31}^-, \mu_{z41}^-, \mu_{z51}^-, \mu_{z61}^-$ )
0.4284167	270, 150, 80	0.989, 0.9077778, 0.15, 0.2366667

Note 2: The table 7 reflects the solution obtained by using the proposed FGP model in terms of achieving the aspired levels of the production goals in the decision making environment.

**Table 7: Comparison of the FGP model solution with the production of Bricks recorded in the year 2012-2013**

Season 1					
Brick type	Soil allocation(V) in the financial year 2012-13	Soil allocation(V) under the proposed model	Production achievements received in year 2012-13	Production achievements under the proposed FGP model	Brick price with carrying cost(1000 pieces)
Brick 1	950	1020	893000	958800	Rs 7500
Brick 2	880	900	827200	846000	Rs 7200
Picket brick	1170	1080	1099800	1015200	Rs 6900
Damaged brick	-	-	180000	180000	Rs 1000
Season 2					
Brick type	Soil allocation(V) in the financial year 2012-13	Soil allocation(V) under the proposed model	Production achievements received in year 2012-13	Production achievements under the proposed FGP model	Brick price with carrying cost(1000 pieces)
Brick 1	190	270	178600	253800	Rs 7400
Brick 2	210	150	197400	141000	Rs 7100
Picket brick	100	80	94000	75200	Rs 6800
Damaged brick	-	-	30000	30000	Rs 1000

Note 3: The table 8 & the table 9 present the total calculation of net and gross profit under the proposed FGP model and net and gross profit obtained from the recorded profit in the financial year 2012-2013.



**Table 8: Profit calculation for the Brick-field for financial year 2012-13**

Season1			Total gross profit (Rs)	Total Payable income tax (RS) in the financial year 2012-2013	Net Profit (Rs) in the financial year 2012-2013
Brick sell(Rs)	Expenditures of the Brick-field(Rs)	Profit(Rs) = Total sell-Total expenditure	Total gross profit (Rs) = Gross profit in season 1(Rs)+ Gross profit in season 2 (Rs)	Total payable income tax (Rs)	Net Profit (Rs)=Gross profit (Rs)- Income tax (Rs)
Brick 1: 6697500.00	Labor: 960000.00	20421960.00 - 14386000.00	6035960.00	2093477.00	7358340.00- 2093477.00
Brick 2: 5955840.00	Machine: 820000.00				
Picket: 7588620.00	Coal: 9260000.00				
Damaged brick:180000.00	Soil purchase: 2400000.00				
	Other expenditure: 946000.00				
Total: 20421960.00	Total: 14386000.00	6035960.00			
Season2			+		
Brick sell (Rs)	Expenditures of the Brick-field(Rs)	Profit(Rs)			
Brick 1: 1321640.00	Labor: 140000.00	3392380.00 - 2070000.00	1322380.00	2093477.00	5264863
Brick 2: 1401540.00	Machine: 140000.00				
Picket: 639200.00	Coal: 1240000.00				
Damaged brick:30000.00	Soil purchase: 400000.00				
	Other expenditure: 150000.00				
Total: 3392380.00	Total: 2070000.00	Total: 1322380.00	Total: 7358340.00	2093477.00	5264863

**Table 9 Profit calculation throughout the financial year under the proposed model**

Season1	Total gross profit in the financial year under the proposed model (Rs)	Payable income tax (Rs) Under the proposed model	Net profit (Rs)



Type of Bricks	Proposed selling of Bricks (Rs)	Expenditures of the Brick-field(Rs)	Total Profit (Rs) = Proposed selling of Bricks (Rs)- Total expenditures of the Brick-field (Rs)	Total gross profit in season 1(Rs)+ Total gross profit in season 2 (Rs)	Payable income tax (Rs)	=Total gross profit- Payable income tax
Brick 1	7191000.00	Labor: 960000.00	20467080.00 -	6081080.00  +  1350580.00	2116132.00	7431660 - 2116132
Brick 2	6091200.00	Machine: 820000.00	14386000.00			
Picket	7004880.00	Coal: 9260000.00				
Damaged brick:	180000.00	Soil purchase: 2400000.00				
		Other expenditure: 946000.00				
Total	20467080.00	14386000.00	6081080.00			
Season2						
Type of Bricks	Proposed selling of Bricks (Rs)	Expenditures of the Brick-field(Rs)	Total Profit (Rs) = Proposed selling of Bricks (Rs)-Total expenditures of the Brick-field (Rs)			
Brick 1	1878120.00	Labor: 140000.00	3420580.00 –	6081080.00  +  1350580.00	2116132.00	7431660.00  2116132.00
Brick 2	1001100.00	Machine: 140000.00	2070000.00			
Picket	511360.00	Coal: 1240000.00				
Damaged brick:	30000.00	Soil purchase: 400000.00				
		Other expenditure: 150000.00				
Total	420580.00	2070000.00	1350580.00			

Note 4: The table 10 reflects that the proposed FGP model offers better optimal solution in gross and net profit income than recorded in the financial data recorded.

**Table 10: Comparison of net and gross profit achievement between the proposed FGP model and net and gross profit recorded in the year 2012-2013**

Profit achievement	Total gross profit (Rs)	Total net profit (Rs)
<b>Profit achievement recorded in the year financial 2012-2013</b>	7358340.00	5264863.00
<b>Profit achievement under the proposed FGP model</b>	7431660.00	5315528.00

Note 5. On comparing Euclidean distance D with Gupta and Bhattacharjee [45], Tiwari et al. [17], Zimmermann [13], the table 11 and table 12 reflect that the proposed FGP model provides the same solution set.



**Table11: Comparisons of solutions by various methods with Euclidean distance function (Season1)**

Methods	Maximizing/ minimizing Function ( $\lambda$ )	( $x_{11}, x_{21}, x_{31}$ ) ( in unit V)	$\mu_{z_{11}}, \mu_{z_{21}}, \mu_{z_{31}},$ $\mu_{z_{41}}, \mu_{z_{51}}, \mu_{z_{61}}$	Euclidean Distance (D)
Proposed FGP approach	Min $\lambda = 2.98176$	1020, 900, 1080	0.9997, 0.7703385, 0.3104615 , 0.326477, 0.3152632 , 0.296	1.556208
M. Gupta and D. Bhattacharjee [45]	Min( $1 - \lambda$ ) = 0.776	1020, 900, 1080	0.9997, 0.7703385, 0.3104615 , 0.326477, 0.3152632 , 0.296	1.556208
Tiwari et al. [17]	Max $\lambda = 3.01824$	1020, 900, 1080	0.9997, 0.7703385, 0.3104615 , 0.326477, 0.3152632 , 0.296	1.556208
Zimmermann [13]	Max $\lambda = 0.296$	1020, 900, 1080	0.9997, 0.7703385, 0.3104615 , 0.326477, 0.3152632 , 0.296	1.556208

**Table12: Comparisons of solutions by various methods/approaches with Euclidean distance function (Season2)**

Methods/Approach	Maximizing/ minimizing Function ( $\lambda$ )	( $x_{12}, x_{22}, x_{32}$ ) ( in unit V)	( $\mu_{z_{12}}, \mu_{z_{22}},$ $\mu_{z_{42}}, \mu_{z_{62}},$ )	Euclidean Distance (D)
Proposed FGP approach	Min $\lambda = 0.4284167$	270, 150, 80	0.989, 0.9077778, 0.15, 0.2366667	1.146213
Tiwari et. all [17]	max $\lambda = 2.286333$	270, 150, 80	0.989, 0.9077778, 0.15, 0.2366667	1.146213
M. Gupta & D. Bhattacharjee [45]	Min ( $1 - \lambda$ ) = 0.4	270, 150, 80	0.989, 0.9077778, 0.15, 0.2366667	1.146213
Zimmermann [3]	max $\lambda = 0.15$	270, 150, 80	0.989, 0.9077778, 0.15, 0.2366667	1.146213



## CONCLUSIONS

The paper presented FGP model for optimal allocation of soil for Brick-fields. It proposed an annual plan for different types of Bricks. The case study showed that owner of Brick-fields would get satisfactory optimal solution if proper soil allocation was done. The proposed approach can be applied in different allocation problems such as land allocation problem in agriculture.

The FGP solution approach to soil allocation problem demonstrated in this paper provides a new basis for analyzing the production achievement of different types of Bricks to the aspired levels.

Under the framework of the proposed model, different other constraints (crisp or fuzzy) can easily be incorporated and proper decision for soil allocation planning can be easily made

This paper firstly deals with soil allocation problem of brick-field. This may open up new field of study in production planning in Brick-fields. The formulation can be extended in fuzzy stochastic environment. .

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